

# Calculation of Population Densities of Helium Atoms in Non-L.T.E. Plasmas

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Particle densities of helium atoms in the ground and excited states have been calculated for non-L.T.E. plasmas on the basis of a collisional-radiative model in which singlet and triplet states have separately been taken into account. Distinction is made between two physical situations: 1— a homogeneous stationary state, 2— a transient and/or inhomogeneous plasma state. In both cases, the particle densities have been calculated for an optically thin, a slightly optically thick and strongly absorbing plasmas. Only the results for the homogeneous stationary state are presented in this paper. Those for the transient and/or inhomogeneous states have been summarized in numerical tables which will be sent on request. (Title: "Tables of reduced population coefficients for the levels of atomic helium", Report EUR-CEA-FC-697.) The tables are sufficiently complete to permit a wide application in the field of spectroscopic diagnostics of different types of non-L.T.E. plasmas. — Comparison of our results with the values measured by Boersch et al.<sup>10</sup> shows good agreement with our calculations when one assumes that the observed plasma is strongly inhomogeneous and dominated by diffusion.

## Introduction

Spectroscopic measurements of line and continuum intensities emitted from astrophysical and laboratory plasmas have shown that the actual emission coefficients and, thus, the population densities deviate very often from those which are predicted theoretically when the observed regions are assumed to be in local thermodynamic equilibrium (L.T.E.) For given values of electron temperature and electron density helium shows the largest deviations from L.T.E. amongst all neutral particles. This is partly due to the large energy gap between the ground and the first excited state, partly due to the special structure of the level system which shows the  $2^3S_0$  (-triplet) state as lowest excited level with a forbidden radiative dipole transition to the ground state  $1^1S_0$  and partly due to the existence of autoionizing states giving rise to dielectronic recombination at high electron temperatures and low electron densities.

Compared to atomic hydrogen the energy differences between many of the lower lying excited levels deviate considerably from the corresponding hydrogen values with same principal quantum number. Moreover, there are considerable differences between the atomic constants of hydrogen and helium. It follows from this that non-L.T.E. cal-

culations of the population densities for atomic hydrogen can — even in scaled form — hardly be used for a calculation of the population densities of helium.

Non-L.T.E. population densities of atomic helium in stationary homogeneous plasmas have already been published some years ago by one of us<sup>1</sup>. The calculations have now been repeated with refined atomic constants such as term values, transition probabilities and cross sections. The present paper summarises the results obtained in a form which allows the calculation of the population densities for different physical situations such as homogeneous stationary, transient and/or inhomogeneous diffusion-dominated helium plasmas.

## Level System and Rate Equations

The level system on which the calculations are based is the same as that one given in Fig. 1 of Ref.<sup>1</sup>, and the levels are characterized by the same subscripts. The particle densities are denoted by the symbols  $X_i$  and  $Y_i$  for the singlet and triplet system respectively:

*Singlet System:*

$N(1^1S)$ $X_0$	$N(2^1S)$ $X_1$	$N(2^1P)$ $X_2$	$N(i \geq 3)$ $X_i$	$N(j)$ $X_j$	$N(p)$ $X_p$
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*Triplet System:*

$N(2^3S)$ $Y_1$	$N(2^3P)$ $Y_2$	$N(i \geq 3)$ $Y_i$	$N(j)$ $Y_j$	$N(p)$ $Y_p$
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The particle densities obey the following rate equations

$$(\partial X_i / \partial t) + \nabla \cdot (X_i \bar{\mathbf{v}}_i^{(X)}) = (\partial X_i / \partial t)_{\text{coll, rad}}, \quad (1a)$$

$$i = 0, 1, 2, 3, \dots, p;$$

$$(\partial Y_i / \partial t) + \nabla \cdot (Y_i \bar{\mathbf{v}}_i^{(Y)}) = (\partial Y_i / \partial t)_{\text{coll, rad}}, \quad (1b)$$

$$i = 1, 2, 3, \dots, p$$

where  $\bar{\mathbf{v}}_i^{(X)}$  and  $\bar{\mathbf{v}}_i^{(Y)}$  denotes the mean diffusion velocity of particles belonging to the  $X$ - or  $Y$ -system respectively. The r.h.s. can be put into the following form

$$(\partial X_i / \partial t)_{\text{coll, rad}} = \sum_{j=0}^p a_{ij} X_j + \sum_{j=1}^p b_{ij} Y_j + \delta_i^{(X)}, \quad (2a)$$

$$i = 0, 1, 2, 3, \dots, p;$$

$$(\partial Y_i / \partial t)_{\text{coll, rad}} = \sum_{j=0}^p c_{ij} X_j + \sum_{j=1}^p d_{ij} Y_j + \delta_i^{(Y)}, \quad (2b)$$

$$i = 1, 2, 3, \dots, p.$$

The coefficients  $a_{ij}$ ,  $b_{ij}$ ,  $c_{ij}$ , and  $d_{ij}$  are the collisional-radiative interaction frequencies,  $\delta_i^{(X)}$  and  $\delta_i^{(Y)}$  are the recombination rates into level  $i$  of the singlet ( $=X$ -) and triplet ( $=Y$ -) system respectively. The analytical expressions for all collision coefficients (calculated for a Maxwellian velocity distribution) and the numerical values of the term values, oscillator strengths and transitions probabilities used in the present calculations may be found in a special report<sup>2</sup>.

A maximum number of 51 levels has been taken into account, i.e. levels up to a principal quantum number  $p = 25$  have assumed to be effective in populating and depopulating the continuum of free electrons. It should be emphasized that for principal quantum numbers  $i \geq 3$  no distinction has been made between the different S, P, D, etc. sublevels of same principal quantum number, i.e. we have assumed that the sublevels belonging to different angular momentum states are populated according to their statistical weights. This is very often a good approximation at high electron densities ( $n_e > 10^{14} \text{ cm}^{-3}$ ), but deviations from the statistical population of the sublevels can occur at low electron densities ( $n_e < 10^{14} \text{ cm}^{-3}$ ) and high electron temperatures in transient and/or diffusion-dominated plasmas. Inspection of the cross sections shows that in this latter case the  $1S$  levels can reach population densities  $N(1S)/1$  which are larger by a factor two to three than the population densities  $N(1P)/3$  of the  $1P$  levels, whereas the  $1D$  levels can be underpopu-

lated by approximately the same factor relative to the  $1P$  levels. The actual population densities depend not only on the collision coefficients but also on the degree of reabsorption of resonance radiation which is much larger for  $1P$ -levels than for all other sublevels.

Radiative absorption has been taken into account by optical escape factors  $A_{ij}$  and  $A_i$  for bound-bound and free-bound radiation respectively. Thus, the spontaneous transition probabilities  $A_{ij}^{(X)}$  and  $A_{ij}^{(Y)}$  have been replaced by  $A_{ij}^{(X)} A_{ij}^{(X)}$  and  $A_{ij}^{(Y)} A_{ij}^{(Y)}$ , and the radiative recombination coefficients  $R_i^{(X)}$  and  $R_i^{(Y)}$  by  $A_i^{(X)} R_i^{(X)}$  and  $A_i^{(Y)} R_i^{(Y)}$  respectively. A transition is optically thin when the corresponding  $A$ -value is equal to one. By putting  $A \neq 1$  it is possible to account for any degree of reabsorption. In principal, the  $A$ -are not independent from each other. For all further details the reader is referred to Reference<sup>2</sup>.

At high electron densities the system of coupled equations has been truncated at a level  $p$  given by the relation  $p = (Z \langle d_+ \rangle / a_0)^{1/2}$  where  $\langle d_+ \rangle$  is the mean distance between charged particles and  $a_0$  the first Bohr radius. For atomic helium  $Z = 1$  holds.

In Ref.<sup>1</sup> we have accounted for atom-atom exchange collisions between singlet and triplet levels. The corresponding interaction terms made the system of coupled rate equations non-linear in the population densities. In the present calculations atom-atom collisions have not been taken into account and this only in order to get a linear system of equations which is necessary for a simple representation of the solutions. The error which is made by dropping atom-atom collision terms is to a large part compensated by the fact that we have now accounted for electron-atom exchange collisions between all levels of the triplet and of the singlet system, symbolically:

$$[\text{electron}] + \left[ \begin{array}{c} \text{any level } i \text{ of} \\ X\text{-system} \end{array} \right] \rightleftharpoons [\text{electron}] + \left[ \begin{array}{c} \text{any level } j \text{ of} \\ Y\text{-system} \end{array} \right].$$

For degrees of ionization larger than  $10^{-3}$ , electron-atom collisions are in any case much more efficient than atom-atom collisions.

Auto-ionization states have not been taken into account. They contribute to the populations of the

\* The superscripts  $(X)$  and  $(Y)$  refer to the  $X$ - and  $Y$ -system respectively.

singly excited levels at low electron densities ( $n_e < 10^{12} \text{ cm}^{-3}$ ) and electron temperatures  $T_e$  larger than approximatively  $5 \cdot 10^4 \text{ K}$ .

In the frame of the assumptions made the values of  $a_{ij}$ ,  $b_{ij}$ ,  $c_{ij}$  and  $d_{ij}$  depend on the following plasma parameters

$$n_e, T_e, A_{ij}^{(X)} \text{ and } A_{ij}^{(Y)},$$

whereas the terms  $\delta_i^{(X)}$  and  $\delta_i^{(Y)}$  are functions of

$$n_+, n_e, T_e, A_i^{(X)} \text{ and } A_i^{(Y)}.$$

### General Form of the Solutions

#### a) Homogeneous stationary state

In this case all time derivatives and all divergences of the diffusion fluxes are equal to zero, i.e.

$$\partial X_i / \partial t = 0, \quad \partial Y_i / \partial t = 0, \quad (3a)$$

$$\nabla \cdot (X_i \bar{\mathbf{v}}_i^{(X)}) = 0, \quad \nabla \cdot (Y_i \bar{\mathbf{v}}_i^{(Y)}) = 0. \quad (3b)$$

The system of rate equations reduces to  $2p+1$  linear coupled equations for the  $2p+1$  levels:

$$0 = \sum_{j=0}^p a_{ij} X_j + \sum_{j=1}^p b_{ij} Y_j + \delta_i^{(X)}, \quad i = 0, 1, \dots, p; \quad (4a)$$

$$0 = \sum_{j=0}^p c_{ij} X_j + \sum_{j=1}^p d_{ij} Y_j + \delta_i^{(Y)}, \quad i = 1, 2, \dots, p. \quad (4b)$$

When  $n_+$ ,  $n_e$ ,  $T_e$ ,  $A_{ij}^{(X)}$ ,  $A_{ij}^{(Y)}$ ,  $A_i^{(X)}$  and  $A_i^{(Y)}$  are given the number densities  $X_i$  and  $Y_i$  can be calculated. They shall be termed *solutions for the homogeneous stationary (HS) state* and may be characterized by the symbols  $X_i^{\text{HS}}$  and  $Y_i^{\text{HS}}$  respectively. The  $X_i^{\text{HS}}$  and  $Y_i^{\text{HS}}$  divided by the corresponding Saha values  $X_i^*$  and  $Y_i^*$  will yield the Saha decrements  $b_i^{(X)}$  and  $b_i^{(Y)}$  for the homogeneous stationary state. They are independent of the ion density  $n_+$ , they depend only on  $n_e$ ,  $T_e$  and the optical escape factors. Especially for the ground level we write

$$b_0^{(X)} \equiv b_0^{\text{HS}} = X_0^{\text{HS}} / X_0^*.$$

For  $n_e \rightarrow \infty$ , the solutions tend to the Saha values, i.e.  $X_i^{\text{HS}} \rightarrow X_i^*$  and  $Y_i^{\text{HS}} \rightarrow Y_i^*$  if  $n_e \rightarrow \infty$ . The Saha values are also obtained for any value of  $n_e$  when the plasma is optically thick in all bound-bound and free-bound transitions (see Ref.<sup>3</sup>). For all other conditions a more or less large number of population densities deviates from the Saha population densities. Especially the ground state will always be overpopulated, i.e.  $X_0^{\text{HS}} > X_0^*$ , whence  $b_0^{\text{HS}} > 1$ .

We consider now a pure helium plasma with  $n_e = n_+$ , i.e. double ionization shall not yet contribute to the total number of electrons. Implicitly we exclude interactions between doubly ionized and neutral helium atoms. To given values of  $n_e = n_e^{\text{HS}}$  and  $T_e = T_e^{\text{HS}}$  and given values of escape factors belongs a distinct value  $X_0 = X_0^{\text{HS}}$  as solution of system (4a, b). The Saha density  $X_0^*$  for the values  $n_e^{\text{HS}}$  and  $T_e^{\text{HS}}$  is given by

$$X_0^* = (n_e^{\text{HS}})^2 \frac{1}{4} [h^3 / (2\pi m_e k T_e)^{3/2}] \exp\{E_0^X / k T_e\}. \quad (5)$$

It is on the other hand possible to calculate from Saha's equation the electron density  $n_e^*$  which belongs to the values  $X_0 = X_0^{\text{HS}}$  and  $T_e = T_e^{\text{HS}}$ :

$$(n_e^*)^2 = X_0^{\text{HS}} 4 [(2\pi m_e k T_e)^{3/2} / h^3] \exp\{-E_0^{(X)} / k T_e\}. \quad (6)$$

We now introduce the ratio

$$b_e^{\text{HS}} = n_e^{\text{HS}} / n_e^* \quad (7)$$

which may be termed "Saha decrement of the electron density" for the homogeneous stationary state. Equations (5) to (7) yield directly the relation between  $b_e^{\text{HS}}$  and  $b_0^{\text{HS}}$ :

$$[b_e^{\text{HS}}(X_0^{\text{HS}}, T_e^{\text{HS}})]^2 = [b_0^{\text{HS}}(n_e^{\text{HS}}, T_e^{\text{HS}})]^{-1}. \quad (8)$$

#### b) Inhomogeneous and/or transient state

In the case of very fast transient phenomena and/or extremely large density or temperature gradients all coupled rate equations (1a, b) have simultaneously to be solved, since temporal and/or spatial relaxation phenomena may occur between a more or less large number of individual levels. For numerical examples see Ref.<sup>4</sup>. Under two conditions, however, the system of coupled equations can considerably be simplified:

(i) After a perturbation of the plasma state (for instance by rising the electron temperature) the excited as well as ground state densities will follow the perturbation and tend to a new state with different relaxation times  $\tau_i$ . Two different kinds of relaxation times have to be distinguished, namely:

1. The relaxation times  $\tau_{i>0}$  for the excited levels to come into a statistical equilibrium with the actual values of electron and ground state particle densities.
2. The relaxation time  $\tau_0$  for the ground level to come into a steady state with the electrons at temperature  $T_e$ .

The values of  $\tau_{i>0}$  are by several orders of magnitude smaller than  $\tau_0$ . For time intervals  $\Delta t$  lying in the limits  $\tau_{i>0} < \Delta t < \tau_0$  the time derivatives of the number densities of excited levels can therefore be considered to be equal to zero provided  $T_e$  and  $n_e$  fulfill the conditions

$$\begin{aligned} (T_e(t + \tau_{i>0}) - T_e(t))/T_e(t) &\ll 1, \\ (n_e(t + \tau_{i>0}) - n_e(t))/n_e(t) &\ll 1, \end{aligned}$$

whereas for time intervals  $\Delta t$  lying between  $\tau_{i>0}$  and  $\tau_0$  the time derivative for the ground state particle density is still different from zero.

(ii) In inhomogeneous plasmas, the situation is similar. Two kinds of relaxation length  $\lambda_i$  have to be considered, namely:

1. The relaxation lengths  $\lambda_{i>0}$  of excited particles to come into a local statistical equilibrium with respect to the local values of electron and ground state particle densities.
2. The relaxation length  $\lambda_0$  for ground state particles to come into statistical equilibrium with respect to the electrons at local temperature  $T_e$ .

The  $\lambda_{i>0}$  are generally very much smaller than  $\lambda_0$ . Under the condition that the smallest lateral dimension of the plasma in direction of the gradients is larger than  $\lambda_{i>0}$  and that for position  $l$  the conditions

$$\begin{aligned} (T_e(l + \lambda_{i>0}) - T_e(l))/T_e(l) &\ll 1, \\ (n_e(l + \lambda_{i>0}) - n_e(l))/n_e(l) &\ll 1 \end{aligned}$$

are fulfilled, the excited particles can be assumed to be already in a quasi-homogeneous state with respect to  $n_e(l)$  and  $X_0(l)$  within dimensions  $\Delta l$  fulfilling  $\lambda_{i>0} < \Delta l < \lambda_0$ , whereas the ground state particles will still deviate from the homogeneous state solution with dimensions  $\Delta l$  lying between  $\lambda_{i>0}$  and  $\lambda_0$ . For regions  $\Delta l$  which are not smaller than  $\lambda_{i>0}$  the divergences of the particle fluxes for the excited particles can therefore be put equal to zero, whereas the divergence of the diffusion flux of ground state particles is still different from zero within dimensions  $\Delta l$  lying between  $\lambda_{i>0}$  and  $\lambda_0$ .

For time and spatial intervals fulfilling the above given conditions the time derivatives of the particle densities and the divergences of the diffusion fluxes of all excited levels can be put equal to zero. This yields  $2p$  coupled equations for the population densities of the  $2p$  excited levels:

$$0 = \sum_{j=0}^p a_{ij} X_j + \sum_{j=1}^p b_{ij} Y_j + \delta_i^{(X)}, \quad i > 0; \quad (9a)$$

$$0 = \sum_{j=0}^p c_{ij} X_j + \sum_{j=1}^p d_{ij} Y_j + \delta_i^{(Y)}, \quad i > 0. \quad (9b)$$

When the values of  $n_+$ ,  $n_e$ ,  $T_e$ , the escape factors and  $X_0$  are given all  $X_{i>0}$  and  $Y_{i>0}$  can be calculated. The solutions can be put into the following mathematical form

$$X_i = X_i^{(0)} + g_i^{(1)} X_0, \quad i > 0; \quad (10a)$$

$$Y_i = Y_i^{(0)} + h_i^{(1)} X_0, \quad i > 0 \quad (10b)$$

where the  $X_i^{(0)}$ ,  $Y_i^{(0)}$  are the solutions of system (9a, b) when all terms containing  $X_0$  are put equal to zero, whereas the  $g_i^{(1)}$ ,  $h_i^{(1)}$  are the solutions of system (9a, b) when all recombination rates  $\delta_i^{(X)}$  and  $\delta_i^{(Y)}$  are put equal to zero and  $X_0$  equal to 1.

The solution for the ground state density,  $X_0$ , follows from the relation

$$\begin{aligned} \partial X_0 / \partial t + \nabla \cdot (X_0 \bar{v}_0^{(X)}) \\ = \sum_{j=0}^p a_{0j} X_j + \sum_{j=1}^p b_{1j} Y_j + \delta_0^{(X)} \end{aligned} \quad (11)$$

which can also be written as follows

$$\partial X_0 / \partial t + \nabla \cdot (X_0 \bar{v}_0^{(X)}) = n_+ n_e \alpha - X_0 n_e S \quad (12)$$

where  $\alpha$  and  $S$  are the collisional-radiative recombination and ionization coefficients. For He-He<sup>+</sup>-e plasmas numerical values of  $\alpha$  and  $S$  may be found in Refs.5,6, for He<sup>+</sup>-He<sup>2+</sup>-e plasmas in Refs.7,8.

When the solutions (10a, b) are divided by the corresponding Saha values  $X_i^*$  and  $Y_i^*$  one obtains the Saha decrements  $b_{i>0}^{(X)}$  and  $b_{i>0}^{(Y)}$  respectively for the excited state populations. The  $b_i$  can be put into the mathematical form

$$b_i = r_i^{(0)} + r_i^{(1)} X_0 / X_0^*, \quad i > 1 \quad (13)$$

where the  $r_i^{(0)}$  and  $r_i^{(1)}$  are different for singlet and triplet levels. For singlet levels:  $r_i^{(0)} = X_i^{(0)} / X_i^*$ ,  $r_i^{(1)} = g_i^{(1)} X_0^* / X_i^*$ . For triplet levels:

$$r_i^{(0)} = Y_i^{(0)} / Y_i^*, \quad r_i^{(1)} = h_i^{(1)} X_0^* / Y_i^*.$$

It is to be noted that these relations permit also the calculation of the excited state populations for the homogeneous stationary state, one has only to put into Eqs. (10a, b) or Eq. (13)  $X_0 = X_0^{\text{HS}}$ .

## The Results

In the following, results are given for three different cases of optical reabsorption, namely a:

- (i) *optically thin plasma*: All photons can freely escape from the plasma, i.e. no reabsorption occurs. All escape factors are equal to unity.

(ii) *slightly optically thick plasma*: It is assumed that reabsorption is only effective in the first members of the resonance series  $j^1P - 1^1S_0$ . Especially

$$A_{02}(X) = 10^{-4}, \quad A_{03}(X) = 10^{-3}, \quad A_{04}(X) = 10^{-2}, \\ A_{05}(X) = 10^{-1},$$

all other  $A_{ij}$  and  $A_i$  assumed to be equal to one. The value  $A_{02}(X) = 10^{-4}$  means for instance that 99,990% of the radiation in the resonance line  $2^1P - 1^1S$  is reabsorbed in the plasma, only  $10^{-4}$  of the mean intensity escapes.

(iii) *strongly absorbing plasma*: In this case

$$A_{02}(X) = 0, \quad A_{03}(X) = 10^{-5}, \quad A_{04}(X) = 10^{-4}, \\ A_{05}(X) = 10^{-3}, \quad A_{06}(X) = 10^{-2}, \quad A_{07}(X) = 10^{-1}, \\ A_{12}(X) = 10^{-2}, \quad A_{13}(X) = 10^{-1}, \\ A_{12}(X) = 10^{-2}, \quad A_{13}(X) = 10^{-1}, \\ A_{23}(X) = 10^{-1}, \quad A_{23}(X) = 10^{-1},$$

and  $A_0(X) = 10^{-2}$  is taken, all other escape factors assumed to be equal to one. The value  $A_{02}(X) = 0$  means that all photons of the resonance line

$2^1P - 1^1S$  are trapped in the plasma. The value  $A_0(X) = 10^{-2}$  means that 99.0% of the photons of the resonance continuum are reabsorbed, only 1% escapes.

#### a) Homogeneous stationary state

Some of the results obtained for the homogeneous stationary state are shown in graphical form in Figures 1 to 6. Figures 1 to 3 give the values of  $X_0^{\text{HS}}$ ,  $n_e^{\text{HS}}$  and  $T_e$  which satisfy system (4a, b) for the three different cases of radiation trapping. From these values one obtains directly the Saha decrements  $b_0^{\text{HS}}$  and  $b_e^{\text{HS}}$ .

— SAHA decrement  $b_0^{\text{HS}}$  for the ground level: When  $n_e$  and  $T_e$  are given,  $X_0^{\text{HS}}$  represents a solution of system (4a, b). This value divided by  $X_0^*$  yields  $b_0^{\text{HS}}$  graphically represented in Figs. 4a–c for resonance trapping according to cases (i), (ii) and (iii). One sees that trapping of resonance photons has a relatively large influence on  $b_0^{\text{HS}}$  for electron densities smaller than  $10^{16} \text{ cm}^{-3}$ . The influence is small for high values of  $n_e$ . (In all three cases  $b_0^{\text{HS}}$  is larger than unity for  $n_e < 10^{17} \text{ cm}^{-3}$ , i.e. complete L.T.E. will only be established when

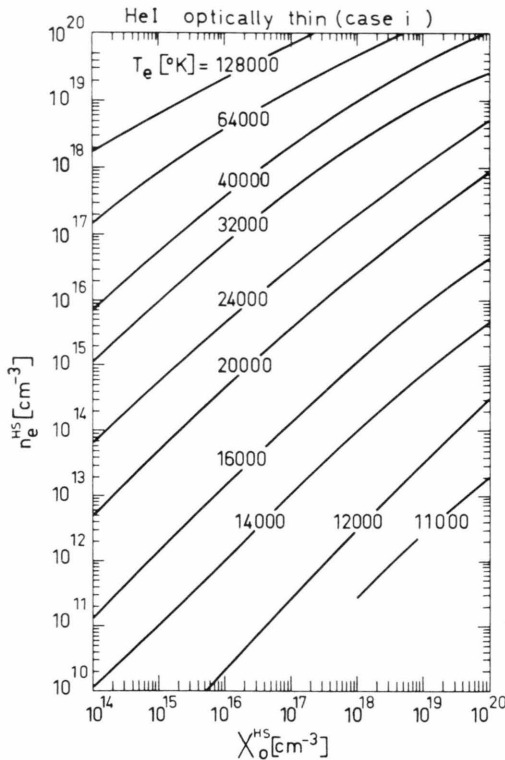


Fig. 1. The solutions  $X_0 = X_0^{\text{HS}}(n_e^{\text{HS}}, T_e)$  of system (4 a, b) for the homogeneous stationary state. All bound-bound and free-bound transitions have assumed to be optically thin according to case (i).

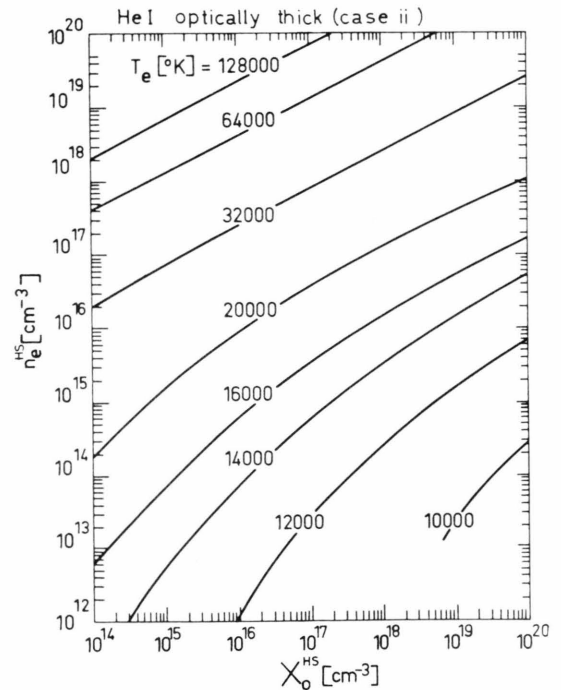


Fig. 2. The solutions  $X_0 = X_0^{\text{HS}}(n_e^{\text{HS}}, T_e)$  of system (4 a, b). Slightly optically thick plasma. Escape factors are according to case (ii).



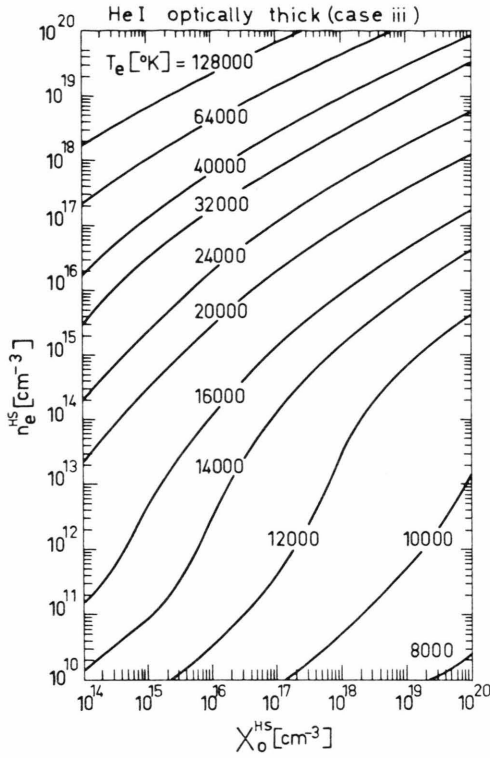


Fig. 3. The solutions  $X_0 = X_0^{\text{HS}}(n_e^{\text{HS}}, T_e)$  of system (4 a, b). Strongly absorbing plasma. Escape factors correspond to case (iii).

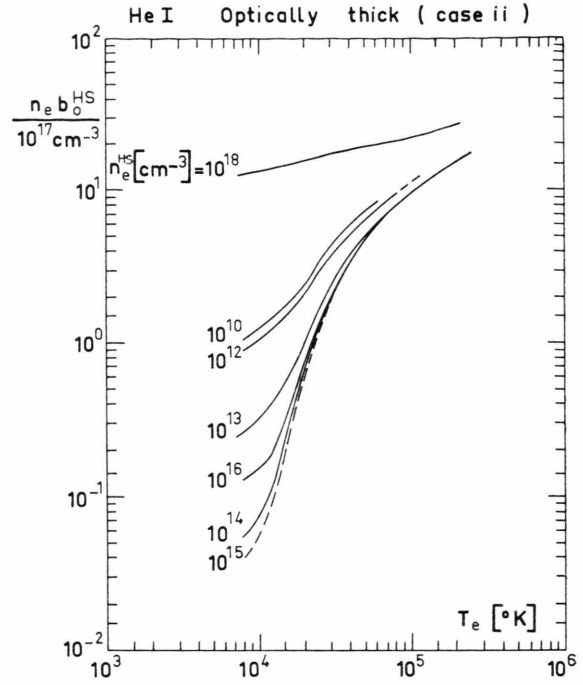


Fig. 4 b

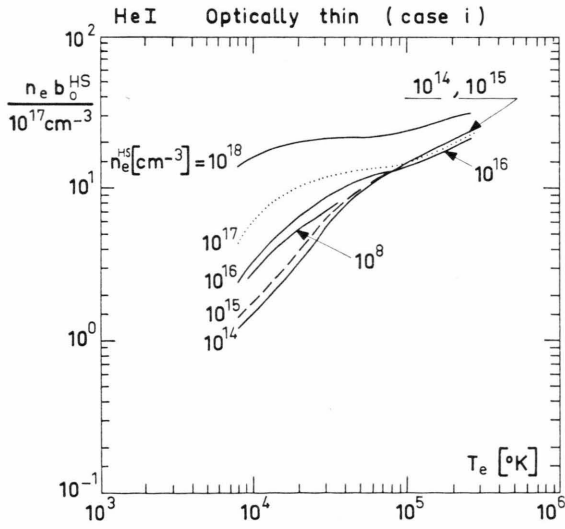


Fig. 4 a

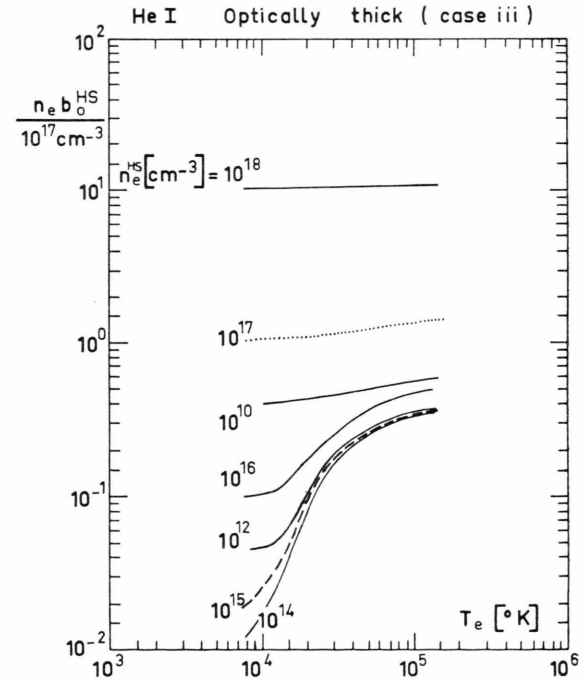


Fig. 4 c

the electron density is larger than  $10^{17} \text{ cm}^{-3}$  (see Fig. 6 for details.)

— SAHA decrement  $b_e^{\text{HS}}$  for the free electrons:  
The values of  $b_e^{\text{HS}}$  as a function of ground state

Fig. 4 a—c. Saha decrement  $b_0^{\text{HS}}$  for the ground level; homogeneous stationary state. a) optically thin plasma (case i), b) slightly optically thick plasma (case ii), c) strongly absorbing plasma (case iii).

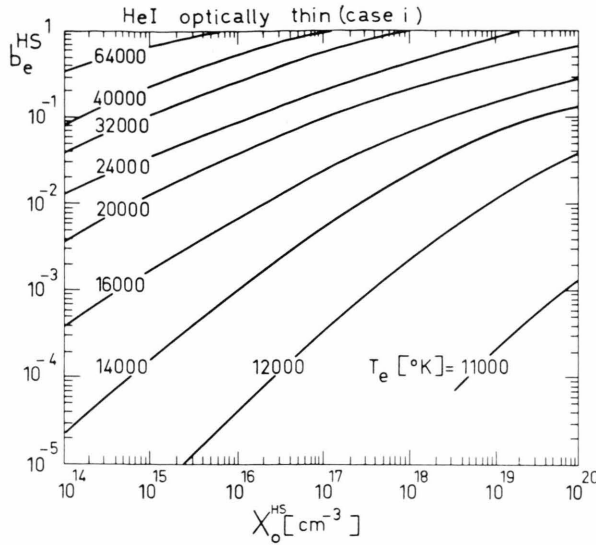


Fig. 5 a

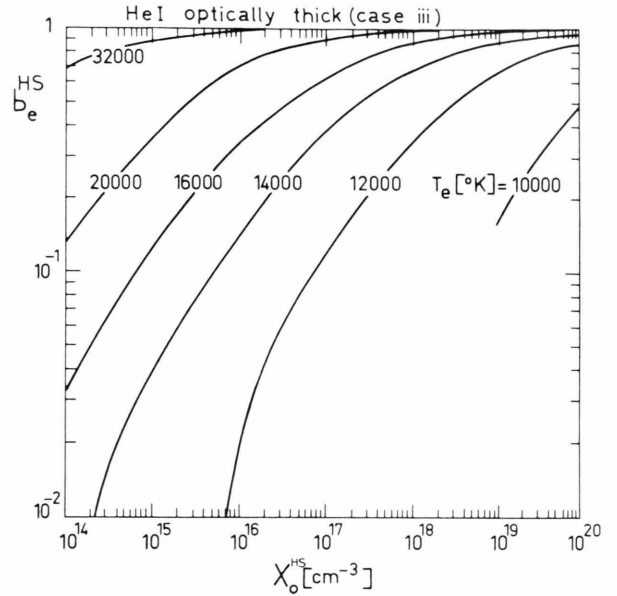


Fig. 5 c

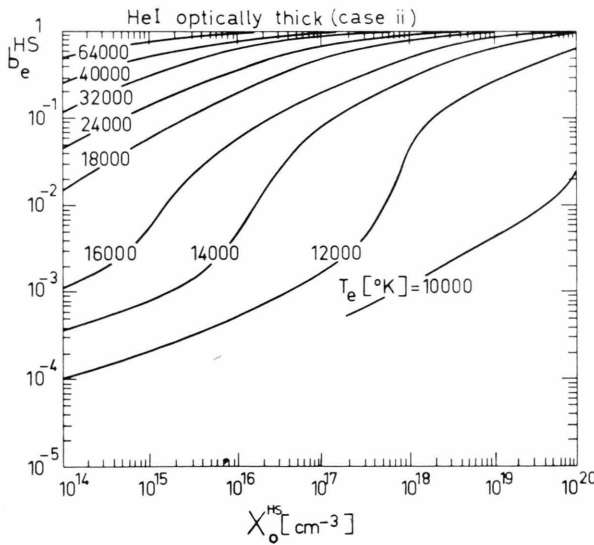


Fig. 5 b

particle density  $X_0^{HS}$  are shown in Figs. 5a—c for the three different cases of reabsorption.  $b_e^{HS}$  is obtained in the following way: For given values of  $A_{ij}$  and  $A_i$ , the system of coupled Eqs. (4a, b) is solved for different pairs of  $X_0$  and  $T_e$ . The solutions are the excited state populations  $X_{i>0}$ ,  $Y_{i>0}$  and the electron density  $n_e^{HS}$ . The values of  $n_e^{HS}$  divided by the Saha values  $n_e^*$  yield  $b_e^{HS}$ .  $b_e^{HS}$  can also be calculated from Equation (8).

The graphical representation exhibits the following features:

Fig. 5 a—c. Saha decrement for the electrons; homogeneous stationary state. a) optically thin plasma (case i), b) slightly optically thick plasma (case ii), c) strongly absorbing plasma (case iii).

1. For a given ground state density  $X_0^{HS}$ ,  $b_e^{HS}$  increases approximately exponentially with temperature until the limit  $b_e^{HS} \rightarrow 1$ . Complete L.T.E. is established when  $b_e^{HS} = 1$  is fulfilled.

2. For given electron temperature,  $b_e^{HS}$  increases with ground state density  $X_0^{HS}$  until the limit  $b_e^{HS} \rightarrow 1$ . The increase is non linear with respect to  $X_0^{HS}$ .

3. Trapping of resonance photons has a considerable influence on  $b_e^{HS}$  at low electron temperatures and low and medium neutral particle densities.

— Establishment of complete L.T.E. “within  $x\%$ ”.

Figure 4a shows that in the optically thin case the ground state particles are still overpopulated (compared to the Saha values) by a factor 1.5 to 3 at an electron density of  $10^{18} \text{ cm}^{-3}$ . In the case of strong reabsorption (case iii, Fig. 4c) the ground state density deviates in any case from the Saha value when  $n_e$  lies below  $10^{17} \text{ cm}^{-3}$ . These high values are due to the fact that the solutions of system (4a, b) tend, at high electron densities, only very smoothly to the L.T.E. limit. The situation changes when one asks: What electron densities are necessary to obtain solutions  $X_0^{HS}$  which deviate

not more than  $x\%$  from the Saha value? (With  $x$  for instance equal to 10, 20, 30 and 50.) The corresponding solutions are shown in graphical form in Figure 6. The lowest curve shows for instance that in the case of strong reabsorption (case iii) the homogeneous stationary state solution for  $X_0$  deviates not more than 50% from the Saha value when  $n_e$  is equal to  $1.7 \cdot 10^{16} \text{ cm}^{-3}$  at a temperature of  $20 \cdot 10^3 \text{ K}$ .

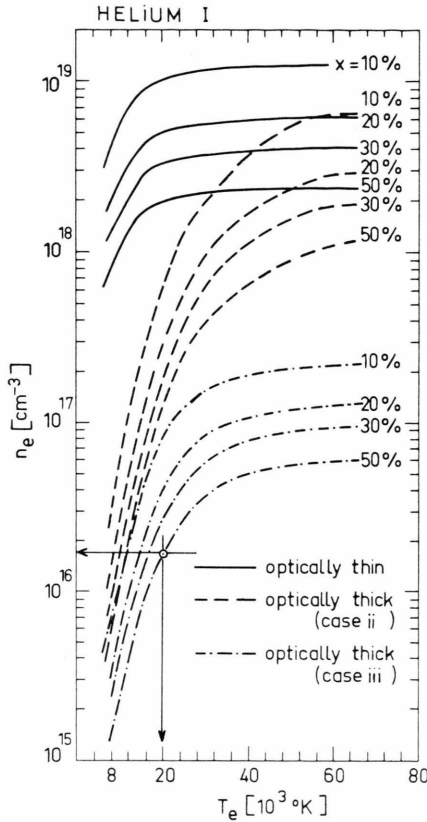


Fig. 6. Electron density  $n_e$  which is necessary to ensure that in a homogeneous stationary non-L.T.E. plasma of temperature  $T_e$  the particle density of helium atoms in the ground state deviates not more than  $x\%$  from the Saha value calculated for the same values of  $n_e$  and  $T_e$ .

#### b) Inhomogeneous and/or transient state

The whole information about the particle densities of excited levels is contained in the coefficients  $r_i^{(0)}$  and  $r_i^{(1)}$ . When these values are known all  $X_{i>0}$  and  $Y_{i>0}$  can be calculated according to Eq. (13) provided the actual number density  $X_0$  of ground state particles is known.  $X_0$  can be determined from either Eqs. (11) or (12) or from any other

relation involving  $X_0$  in suitable form. The relation for the total pressure

$$p = X_0 k T_0 + \sum_{i>0} (X_i + Y_i) k T_0 + n_e k T_e \quad (14)$$

is for instance such a relation.

The numerical values of  $r_i^{(0)}$  and  $r_i^{(1)}$  for different escape factors  $A_{ij}^{(X)}$  and  $A_{ij}^{(Y)}$  have been listed in a number of tables summarized in a special report<sup>9</sup> which will be sent on request. It should be noted that the  $r_i^{(0)}$  and  $r_i^{(1)}$  are independent of the escape factor  $A_0^{(X)}$ .

$A_0^{(X)}$  intervenes only in the calculation of the recombination coefficient  $\alpha$  and, thus, in the calculation of  $X_0$  from Eqs. (11) or (12).

According to the present model, all levels of principal quantum number  $i \geq 3$  have been assumed to be hydrogen-like with a statistical population of the S, P, D, ... sublevels. The singlet levels with  $i \geq 3$  have accordingly statistical weights  $g_i^{(X)}$  equal to  $i^2$ , the triplet levels statistical weights  $g_i^{(Y)}$  equal to  $3i^2$ . These values have to be used when the population densities  $X_i$  and  $Y_i$  are calculated from Equation (13):

$$X_i = b_i^{(X)} X_i^*, \quad Y_i = b_i^{(Y)} Y_i^*. \quad (15)$$

The statistical weights intervene via the Saha values  $X_i^*$  and  $Y_i^*$ . The sublevels are populated according to the ratios 1 : 3 : 5 : 7 ... The number density of the  $i^3D$  level is for instance given by  $N(i^3D) = (15/3 i^2) Y_i = (5/i^2) Y_i$ . For principal quantum numbers  $i < 3$  one has to use the statistical weights according the relation  $(2S + 1)(2L + 1)$ .

#### An Application

The results shall now be applied to a stationary capillary He-arc discharge recently studied by Boersch et al.<sup>10</sup> The discharge was operated in a tube of 3.5 mm inner diameter, the total discharge current was 5 A at a total pressure of 0.7 Torr. The measured particle densities have been listed in Table 1, column I. According to these authors, the electron density should be correct within  $\pm 50\%$ , the ground state density (not directly measured but estimated from the total pressure) can be wrong by a factor of approximately two. The authors have tried to calculate a temperature  $T_e$  from Saha's equation, it was impossible to find solutions which are consistent with the measured values (see Table 2



Table 1. Comparison of calculated with measured population densities (in units  $\text{cm}^{-3}$ ) of neutral helium atoms. The electron temperature  $T_e$  is in  $^\circ\text{K}$ , the electron density  $n_e$  in  $\text{cm}^{-3}$ .

Level	I Experm. <sup>10</sup> $T_e = ?$	II Inhomogeneous stationary state model <sup>a</sup>		III Homogeneous stationary state model					
		$T_e = 32\,000$		$T_e = 64\,000$	$T_e = 16\,000$		$T_e = 32\,000$	$T_e = 64\,000$	
1 1S	$n_e \approx 4 \cdot 10^{12}$	$n_e = 10^{12}$	$n_e = 10^{13}$	$n_e = 10^{12}$	$n_e = 10^{13}$	$n_e = 10^{12}$	$n_e = 10^{12}$	$n_e = 10^{13}$	$n_e = 10^{13}$
2 1S	$5.5 \cdot 10^{15b}$	$5.5 \cdot 10^{15}$	$5.5 \cdot 10^{15}$	$4.7 \cdot 10^{14}$	$1.7 \cdot 10^{15}$	$5.3 \cdot 10^{10}$	$3.5 \cdot 10^{11}$	$4.4 \cdot 10^8$	$3.5 \cdot 10^9$
2 1S	$1.0 \cdot 10^{11}$	$3.7 \cdot 10^{10}$	$5.7 \cdot 10^{11}$	$2.5 \cdot 10^{12}$	$1.2 \cdot 10^{12}$	$1.0 \cdot 10^8$	$1.2 \cdot 10^6$	$6.4 \cdot 10^6$	$1.2 \cdot 10^6$
2 1P	$4.8 \cdot 10^{10}$	$8.8 \cdot 10^{10}$	$8.3 \cdot 10^{10}$	$1.3 \cdot 10^{12}$	$2.0 \cdot 10^{12}$	$5.2 \cdot 10^{12}$	$5.4 \cdot 10^5$	$9.9 \cdot 10^6$	$2.1 \cdot 10^6$
3 1P	—	$1.1 \cdot 10^8$	$8.5 \cdot 10^8$	$5.6 \cdot 10^9$	$4.2 \cdot 10^{10}$	$1.7 \cdot 10^3$	$6.2 \cdot 10^2$	$4.0 \cdot 10^4$	$1.7 \cdot 10^4$
4 1P	—	$3.1 \cdot 10^7$	$1.8 \cdot 10^8$	$3.7 \cdot 10^9$	$1.1 \cdot 10^{10}$	$4.0 \cdot 10^2$	$1.4 \cdot 10^2$	$1.1 \cdot 10^4$	$4.2 \cdot 10^3$
2 3S	$2.4 \cdot 10^{11}$	$1.9 \cdot 10^{11}$	$4.5 \cdot 10^{11}$	$1.9 \cdot 10^{13}$	$4.0 \cdot 10^{12}$	$9.4 \cdot 10^8$	$3.1 \cdot 10^7$	$7.7 \cdot 10^7$	$6.8 \cdot 10^6$
2 3P	$2.0 \cdot 10^{11}$	$7.1 \cdot 10^{10}$	$1.6 \cdot 10^{11}$	$1.0 \cdot 10^{12}$	$1.7 \cdot 10^{12}$	$1.6 \cdot 10^7$	$1.1 \cdot 10^6$	$2.3 \cdot 10^7$	$3.0 \cdot 10^6$
3 3P	—	$1.2 \cdot 10^8$	$1.0 \cdot 10^9$	$4.7 \cdot 10^9$	$2.9 \cdot 10^{10}$	$2.2 \cdot 10^3$	$6.6 \cdot 10^2$	$7.0 \cdot 10^4$	$2.1 \cdot 10^4$
4 4P	—	$5.2 \cdot 10^7$	$2.6 \cdot 10^8$	$2.4 \cdot 10^9$	$9.1 \cdot 10^9$	$8.3 \cdot 10^2$	$3.0 \cdot 10^2$	$2.7 \cdot 10^4$	$8.8 \cdot 10^3$

<sup>a</sup> Estimated, possible error 100%. The gas temperature  $T_0$  is estimated to 1500  $^\circ\text{K}$ .

<sup>b</sup> The ground state particle density ( $5.5 \cdot 10^{15} \text{ cm}^{-3}$ ) is now a given quantity.

of Ref.<sup>10</sup>). Either the ground state or the excited state populations deviated by many orders of magnitude from the measured values. The measured data were then compared with non-L.T.E. calculations published in Ref.<sup>1</sup> for an optically thin homogeneous stationary state plasma. An agreement between measured and calculated values could not be obtained, the discrepancies remained extremely large (six orders of magnitude for the ground state density, two orders of magnitude for the density of excited levels). The reasons for these discrepancies lie in the fact that the observed plasma was neither in L.T.E. nor was it an optically thin homogeneous stationary non-L.T.E. plasma. In the contrary, strong diffusion fluxes occurred and, thus, influenced the thermodynamic state. The measurements should therefore be compared with a non-L.T.E. model containing diffusion terms due to transport of particles across the plasma column and in which the optical thickness is also taken into account.

Helium spectral lines broadened by Doppler and Stark effects show Voigt profiles. The optical escape factors for Voigt profiles can be calculated from the formulas given in Ref.<sup>11</sup>. According to the experimental conditions given in Ref.<sup>10</sup> the mean escape length for photons is approximately equal to 1.5 mm, the gas temperature approximately equal to 1500  $^\circ\text{K}$ , the electron density equal to  $4 \cdot 10^{12} \text{ cm}^{-3} \pm 50\%$ . With these values it is possible to calculate mean values for the optical escape factors, since the ground state density  $X_0$  can now be estimated from the total pressure  $p = 0.7$  Torr. The pressure equation replaces in the present model the diffusion Eq. (12) for the calculation of the ground state density. One obtains  $X_0 = N(1^1S_0) \approx 5.5 \cdot 10^{15} \text{ cm}^{-3}$  (see also Ref.<sup>10</sup>). The electron temperature is still unknown. It can be determined by solving the coupled system of rate equations for different values of  $T_e$ . For a distinct temperature value calculated population densities should agree with the measured data.

Application of the inhomogeneous stationary state model yields particle densities as listed in column II of Table 1. In the electron density range  $n_e = 10^{12} \dots 10^{13} \text{ cm}^{-3}$  good agreement between measured and calculated values is obtained for  $T_e \approx 32\,000 \text{ }^\circ\text{K}$ . For  $T_e < 30\,000 \text{ }^\circ\text{K}$  the calculated values are too low, for  $T_e > 36\,000 \text{ }^\circ\text{K}$  are too high compared to the measured ones. A temperature of  $T_e \approx 20\,000 \text{ }^\circ\text{K}$  as estimated in Ref.<sup>10</sup> is obviously too low.

We have also listed the particle densities for the homogeneous stationary state. In this case the system is entirely determined when  $n_e$  and  $T_e$  are given. In practice an iteration procedure is necessary, since optical escape factors — which depend on the particle densities — can not be calculated without knowing the particle densities. Column III

of Table 1 gives the particle densities calculated for the homogeneous stationary state. It is impossible to find in the electron density range  $10^{12} \dots 10^{13} \text{cm}^{-3}$  a temperature value such that the calculated particle densities agree with the measured ones. This example shows that only a diffusion non-L.T.E. model can explain the measurements.

<sup>1</sup> H. W. Drawin, Z. Naturforsch. **19 a**, 1451 [1964].

<sup>2</sup> H. W. Drawin and F. Emard, Instantaneous Population Densities of the Excited Levels of Hydrogen Atoms, Hydrogen-like Ions, and Helium Atoms in Optically Thin and Thick non-L.T.E. Plasmas, Report EUR-CEA-FC-534, Fontenay-aux-Roses 1970.

<sup>3</sup> H. W. Drawin, Z. Naturforsch. **24 a**, 1492 [1969].

<sup>4</sup> C. C. Limbaugh and A. A. Mason, Phys. Rev. A **4**, 2368 [1971].

<sup>5</sup> H. W. Drawin and F. Emard, Z. Physik **243**, 326 [1971].

<sup>6</sup> H. W. Drawin and F. Emard, Z. Physik **254**, 202 [1972]; **256**, 96 [1972].

<sup>7</sup> R. Mewe, Z. Naturforsch. **25 a**, 1798 [1970].

<sup>8</sup> H. W. Drawin, Z. Physik **253**, 100 [1972] and Report EUR-CEA-FC-642, Fontenay-aux-Roses 1972.

<sup>9</sup> H. W. Drawin and F. Emard, Tables of Reduced Population Coefficients for the Levels of Atomic Helium, Report EUR-CEA-FC-697, Fontenay-aux-Roses 1973.

<sup>10</sup> H. Boersch, J. Geiger, and M. Topschowsky, Z. Naturforsch. **26 a**, 198 [1971].

<sup>11</sup> H. W. Drawin and F. Emard, Beiträge aus der Plasma-physik **13**, Heft N° 3 [1973], in the press.